

- Du Mouvement Politique en France depuis 1789 jusqu'à nos jours. 8vo. *Toulon* 1869. The Author.
- The Mortality Experience of Life Assurance Companies, collected by the Institute of Actuaries. 8vo. *London* 1869. The Institute.
- The New System of Astronomy ; or, is the Earth a Fixed Star or Planet ? 8vo. *London* 1869. The Author.
- The True Theory of the Earth, and Philosophy of the Predicted End. 8vo. *Edinburgh* 1869. The Author.

The following communications were read :—

- I. "Researches into the Constitution of the Opium Bases.—Part III. On the Action of Hydrochloric Acid on Codeia." By AUGUSTUS MATTHIESSEN, F.R.S., Lecturer on Chemistry in St. Bartholomew's Hospital, and C. R. A. WRIGHT, B.Sc. Received July 23, 1869. (See p. 83.)
- II. "On the Thermodynamic Theory of Waves of Finite Longitudinal Disturbance:" and Supplement. By W. J. MACQUORN RANKINE, C.E., LL.D., F.R.SS. Lond. & Edinb. Received August 13, 1869. (See p. 80.)
- III. "On Abstract Geometry." By PROFESSOR CAYLEY. Received October 14, 1869. (Abstract.)

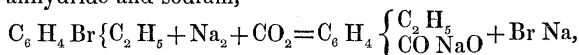
I submit to the Society the present exposition of some of the elementary principles of an Abstract m -dimensional geometry. The science presents itself in two ways,—as a legitimate extension of the ordinary two- and three-dimensional geometries ; and as a need in these geometries and in analysis generally. In fact whenever we are concerned with quantities connected together in any manner, and which are, or are considered as variable or determinable, then the nature of the relation between the quantities is frequently rendered more intelligible by regarding them (if only two or three in number) as the coordinates of a point in a plane or in space ; for more than three quantities there is, from the greater complexity of the case, the greater need of such a representation ; but this can only be obtained by means of the notion of a space of the proper dimensionality ; and to use such representation, we require the geometry of such space. An important instance in plane geometry has actually presented itself in the question of the determination of the curves which satisfy given conditions : the conditions imply relations between the coefficients in the equation of the curve ; and for the better understanding of these relations it was expedient to consider the coefficients as the coordinates of a point in a space of the proper dimensionality.

A fundamental notion in the general theory presents itself, slightly in

plane geometry, but already very prominently in solid geometry; viz. we have here the difficulty as to the form of the equations of a curve in space, or (to speak more accurately) as to the expression by means of equations of the twofold relation between the coordinates of a point of such curve. The notion in question is that of a k -fold relation,—as distinguished from any system of equations (or onefold relations) serving for the expression of it,—and giving rise to the problem how to express such relation by means of a system of equations (or onefold relations). Applying to the case of solid geometry my conclusion in the general theory, it may be mentioned that I regard the twofold relation of a curve in space as being completely and precisely expressed by means of a system of equations ($P=0, Q=0, \dots T=0$), when no one of the functions $P, Q, \dots T$, as a linear function, with constant or variable *integral* coefficients, of the others of them, and when *every surface whatever* which passes through the curve has its equation expressible in the form $U=AP+BQ \dots +KT$, with constant or variable integral coefficients, $A, B \dots K$. It is hardly necessary to remark that all the functions and coefficients are taken to be rational functions of the coordinates, and that the word *integral* has reference to the coordinates.

IV. "On the Action of Bromine upon Ethylbenzol." By T. E. THORPE, Ph.D. Communicated by H. E. ROSCOE, Ph.D. Received November 11, 1869.

In the course of an investigation upon ethylbenzoic acid which Prof. Kekulé and I recently published in conjunction, we had occasion to prepare a quantity of monobromethylbenzol, $C_6H_4Br\{C_2H_5\}$. Our object in this research was to prove experimentally the identity of the ethylbenzoic acid made synthetically by acting upon the monobromethylbenzol by means of carbonic anhydride and sodium,



with the acid subsequently obtained by Fittig by oxidizing diethylbenzol, $C_6H_4\left\{\begin{matrix} C_2H_5 \\ C_2H_5 \end{matrix}\right\}$, by means of nitric acid.

In the preparation of the bromide for the purposes of our experiments, we followed the direction given by Fittig and König, by whom this substance was first described. Bromine was added drop by drop to well-cooled ethylbenzol in the proportion of 1 mol. bromine to 1 mol. ethylbenzol, and the mixture was allowed to stand one or two days before distillation. The action of bromine upon ethylbenzol is extremely energetic, each drop of the bromine disappears almost immediately on coming in contact with the hydrocarbon, the mixture, unless carefully cooled, becomes very hot, and large quantities of hydrobromic acid are evolved. It is easy to perceive when the proper point in the substitution is reached, since after the addition of the theoretical quantity of bromine in order to form $C_6H_4Br\{C_2H_5\}$,